Linear Programming

Multiple Choice Questions

Q: 1 **Shown below is a linear programming problem (LPP).**

Maximise Z = $x + y$

subject to the constraints:

$x + y \leq 1$
$-3x + y \geq 3$
$x \geq 0$
$y \geq 0$

Which of the following is true about the feasible region of the above LPP?

1. It is bounded.
2. It is unbounded.
3. There is no feasible region for the given LPP.
4. (cannot conclude anything from the given LPP)

Q: 2 **Two statements are given below - one labelled Assertion (A) and the other labelled Reason (R). Read the statements carefully and choose the option that correctly describes statements (A) and (R).**

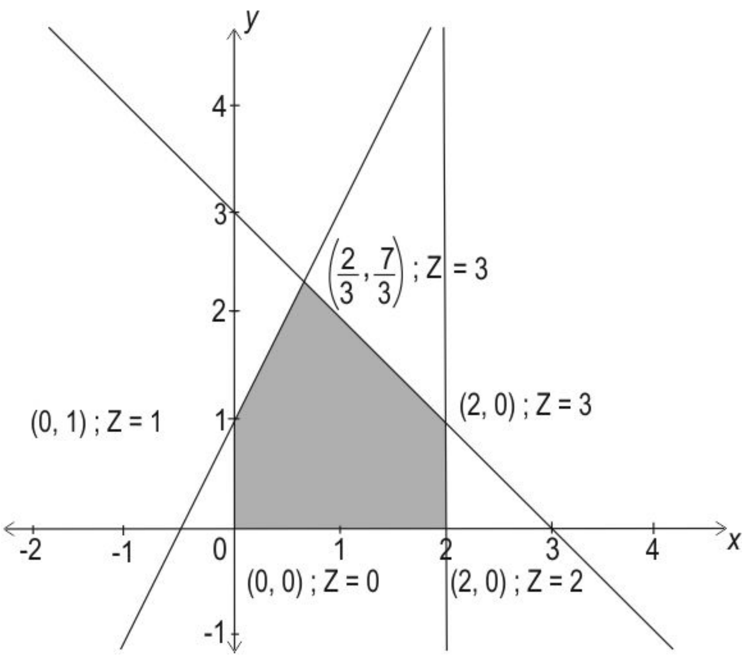
***Assertion* (*A*): All the points in the feasible region of a linear programming problem are optimal solutions to the problem.**

***Reason* (*R*): Every point in the feasible region satisfies all the constraints of a linear programming problem.**

1. Both (A) and (R) are true and (R) is the correct explanation for (A).
2. Both (A) and (R) are true but (R) is not the correct explanation for (A).
3. (A) is false but (R) is true.
4. Both (A) and (R) are false.

Free Response Questions

Q: 3 **Shown below is the feasible region of a linear programming problem(LPP) whose objective function is maximize Z = x + y.** **[1]**



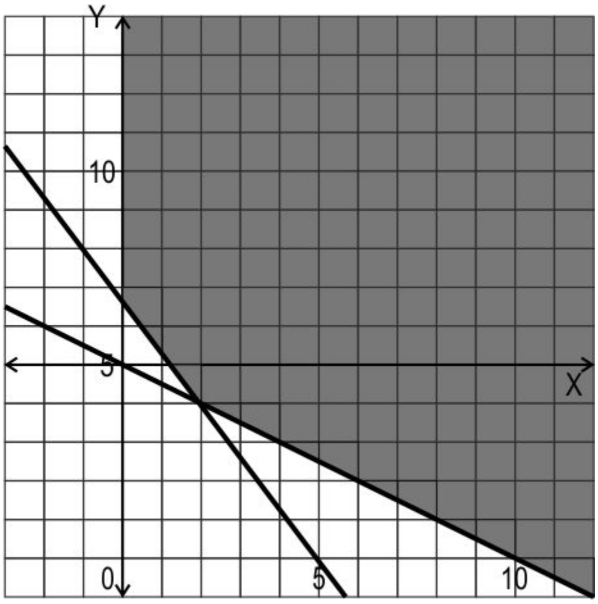
Sarla claimed that there exists no optimal solution for the LPP as there is no unique maximum value at the corner points of its feasible region.

Is her claim true? Give a valid reason.

Q: 4 **State whether the following statement is true or false. Explain your reasoning.** **[1]**

In a linear programming problem, it is possible for a non-corner point to have the optimal value of the objective function, *Z* = a *x* + b *y*.

Q: 5 **The shaded region in the graph shown below represents the feasible region for a linear programming problem with objective function Minimize: Z = 4 *x* - 3 *y*.** **[1]**

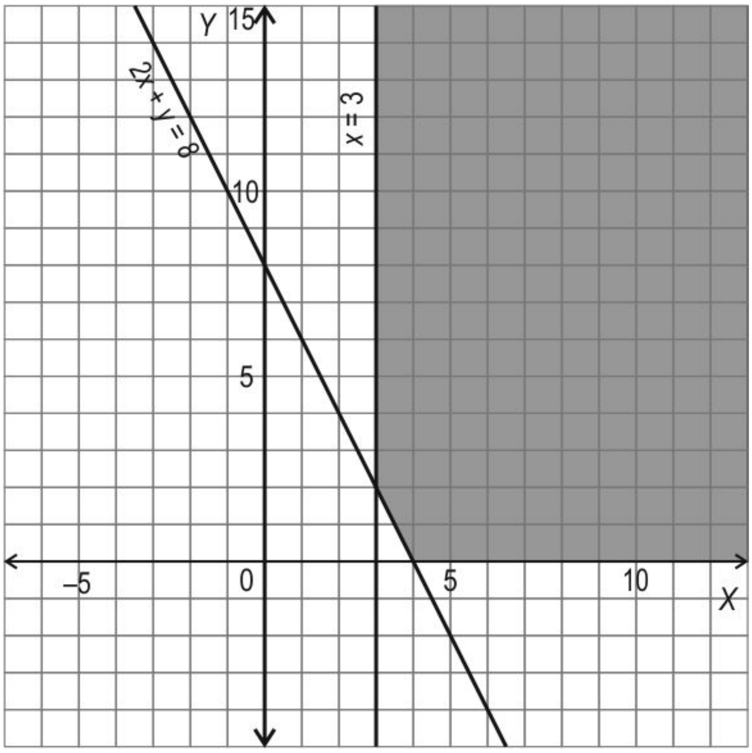


The values of the objective function at the corner points are given below:

Corner points	$Z = 4x - 3y$
$(0, \frac{20}{3})$	-20
$(2, 4)$	-4
$(10, 0)$	40

Suhas says that the minimum value of Z is (-20). Is he correct or incorrect? Justify your answer.

Q: 6 **A feasible region with respect to certain constraints is shaded in the graph below.** **[1]**



Reece says that the objective function Z = $x + y$ will give an optimal solution when maximized. Is he correct or incorrect? Justify your answer.

Q: 7 **The objective function of a linear programming problem is given by:** **[2]**

Maximise Z = $ax + by$

The following are known about the linear programming problem:

♦ **The problem has an unbounded feasible region.**

♦ **(x_0, y_0) is a corner point of the feasible region such that $ax_0 + by_0$ has the maximum value among all the values of Z evaluated at the corner points of the feasible region.**

♦ **The optimal solution of this problem does not exist.**

When and why does this occur?

Q: 8 **Benjamin, a professional runner, follows a rigorous workout routine designed to improve his athletic performance. Running on the treadmill is a part of his daily routine.** **[3]**

If he runs at 10 km/hr, he burns 90 cal/km. If he runs at a faster speed of 16 km/hr, the calories burnt increases to 150 cal/km. He wishes to run maximum distance in not more than an hour at only two speeds. He does not want to burn more than 1000 calories as it could be detrimental for his health.

Express the above optimisation problem as a linear programming problem.

(Note: Treadmill is a device used for exercise, consisting of a continuous moving belt on which one can walk or run.)

Q: 9 **A government bond or sovereign bond is a form of bond issued by the government to raise money for public works such as parks, libraries, bridges, roads, and other infrastructure. There are multiple variants of bonds issued by Government of India (GOI) at fixed interest rates and for a fixed period.** **[5]**

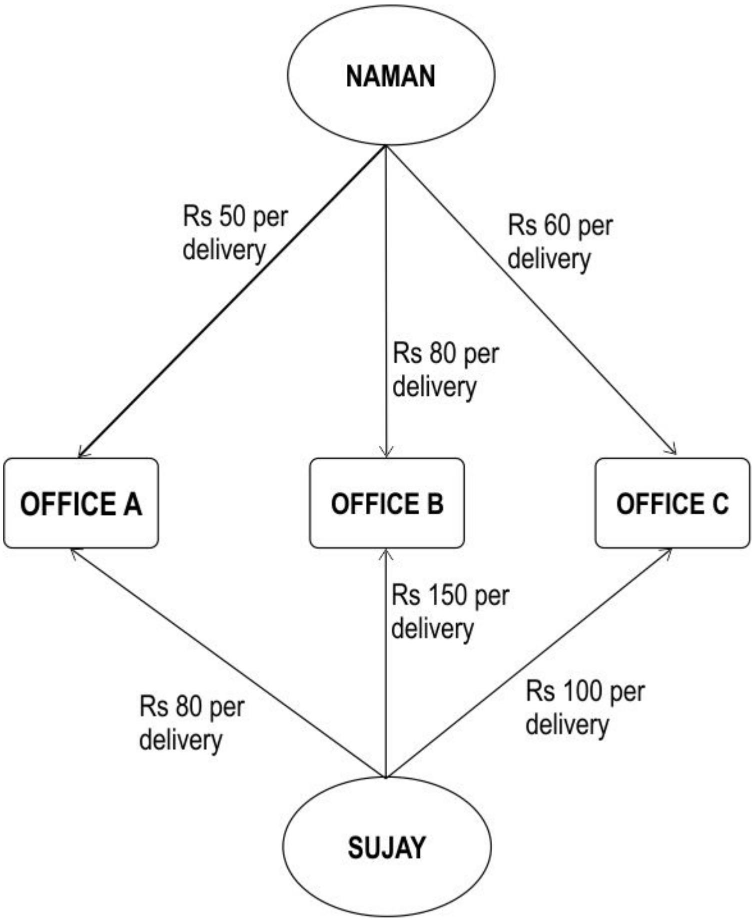
Jayesh Runjhunwala, an investor, wants to invest a maximum of Rs 20000 in government bonds for one year. He decides to invest in two types of bonds X and Y, bond X yielding 10% simple interest on the amount invested and bond Y yielding 15% simple interest on the amount invested. He wants to invest at least Rs 5000 in bond X and no more than Rs 8000 in bond Y.

Formulate the linear programming problem and determine the amount he must invest in two bonds to maximise his return. Show your work.

Q: 10 Naman and Sujay work for Megha's tiffin service as delivery people. In their area, there are 3 offices that order lunch from Megha's tiffin service. Naman and Sujay's daily targets are to deliver 30 and 40 tiffins, respectively. They can each carry only one tiffin at a time to deliver to an office, to prevent the food from spilling out. [5]

On average, the number of tiffins ordered by offices A, B and C daily are 20, 30 and 20 respectively.

The cost of travelling to/from each office to deliver a tiffin is shown in the tree diagram below:



Megha wants to minimise the cost of delivery for her tiffins.

i) Using the given tree diagram, form a system of inequalities to represent the above situation, in order to find out how many tiffins Naman and Sujay should each deliver to a respective office in a day.

ii) Find the function of the daily cost of delivery for Megha's tiffin service.

iii) Graphically solve the system of inequalities obtained in step i) to minimise the daily cost of delivery.

Show your work.

Q: 11 Sudhanva has two kinds of areca nuts - premium and economy. He sells them to two stores, A and B. The profit margin per kg for premium and economy nuts are Rs 80 and Rs 50 respectively. **[5]**

Both stores buy an equal quantity of premium nuts. However, Store B buys three times the economy nuts as compared to Store A. Store A can buy a maximum of 30 kg of nuts and store B can buy a maximum of 60 kg of nuts.

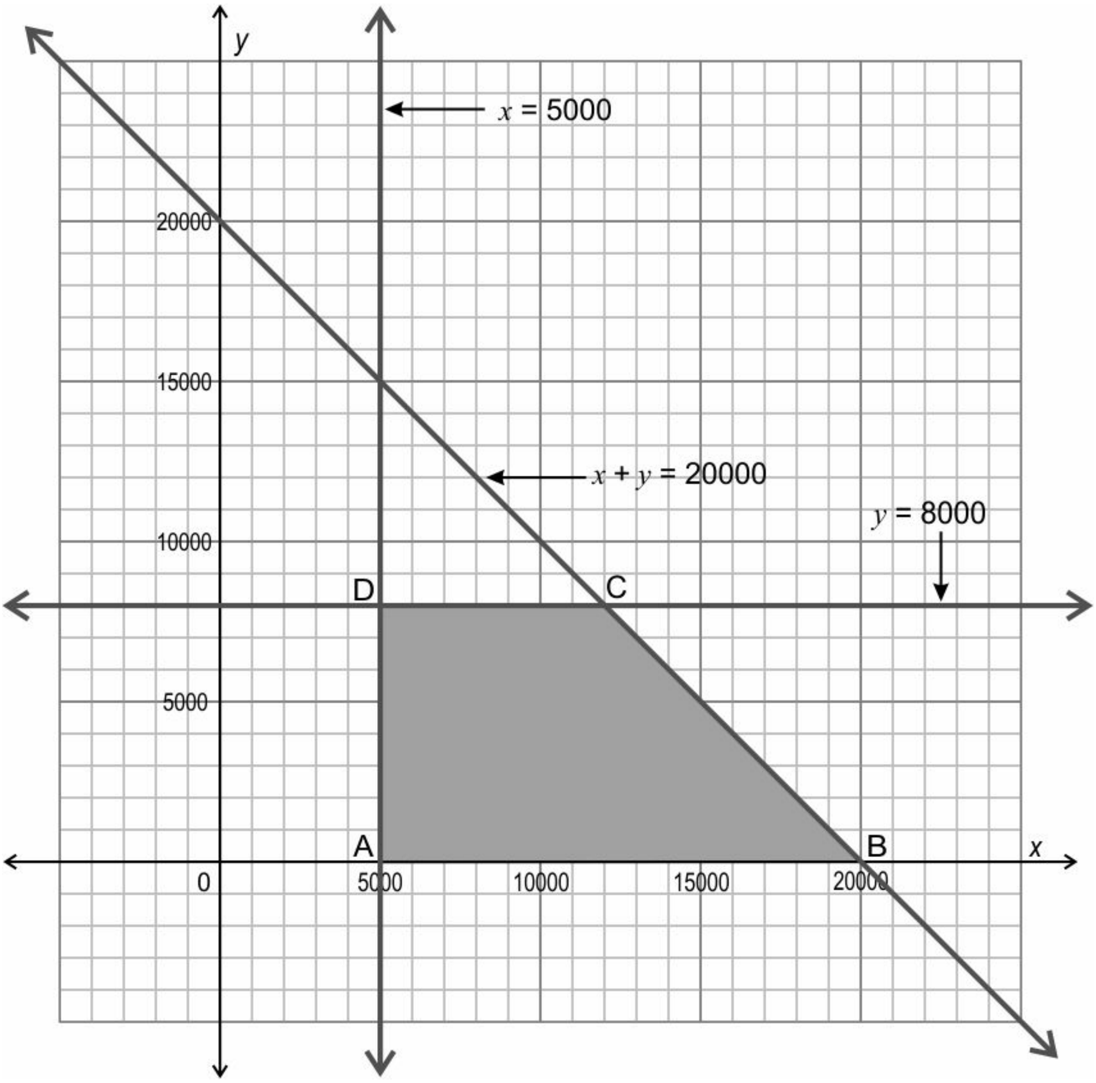
What should be the maximum weight of premium and economy nuts that Sudhanva should sell to obtain maximum profit? Show your steps.

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	3
2	3

Q.No	What to look for	Marks
3	Writes that Sarla's claim is false.	0.5
	Gives a reason. For example, every point on the line joining ($\frac{2}{3}$, $\frac{7}{3}$) and (2, 1) gives the maximum value of Z which is 3. Hence, any point on the line joining ($\frac{2}{3}$, $\frac{7}{3}$) and (2, 1) is an optimal solution.	0.5
4	Writes that the statement is true.	0.5
	Gives a valid reason. For example, if two adjacent corner points yield the optimal value for the objective function, then every point on the line joining them also yields the optimal value.	0.5
5	Writes that Suhas is incorrect.	0.5
	Reasons that since the feasible area is unbounded, the minimum value of the objective function will exist only if the graph of $4x - 3y < (-20)$ has no point in common with the feasible region. Shows as an example that (1, 10) satisfies the inequality $4x - 3y < (-20)$ and is in common with the feasible region and hence, the minimum value of Z does not exist.	0.5
6	Writes that he is incorrect.	0.5
	Finds the value of the objective function at the corner points (2, 3) and (4, 0) as 5 and 4 respectively, and writes that $x + y > 5$ is overlapping with the feasible region and hence, no maximum value exists.	0.5
7	Writes that when $Z > ax_0 + by_0$ has at least one point common with the feasible region, the optimal solution of this problem does not exist.	0.5
	Gives reason that when $Z > ax_0 + by_0$ has at least one point common with the feasible region, it means that there is at least one non-corner point (x_1, y_1) in the feasible region such that $ax_1 + by_1 > ax_0 + by_0$.	1
	Further reasons that Z has no maximum value at (x_0, y_0) and thus the optimal solution of this problem does not exist as a maximum value, if exists, has to be obtained at a corner point .	0.5

Q.No	What to look for	Marks
8	**Takes *x* and *y* to be the distances (in km) covered by Benjamin at the speeds of 10 km/hr and 16 km/hr respectively.** **Finds the time consumed in covering these distances as $\frac{x}{10}$ hr and $\frac{y}{16}$ hr respectively.**	**1**
	Writes the objective function of the given LPP as: **Maximise Z = $x + y$, where Z is the total distance covered.**	**0.5**
	Finds the calorie constraint corresponding to the given LPP as: **$90x + 150y \leq 1000$**	**0.5**
	Finds the time constraint corresponding to the given LPP as: $\frac{x}{10} + \frac{y}{16} \leq 1$	**0.5**
	Writes the non-negativity constraints corresponding to the given LPP as: $x \geq 0$ $y \geq 0$	**0.5**
9	**Assumes the amount to be invested in bonds X and Y to be Rs *x* and Rs *y* respectively.** **Frames the objective function of the given problem as:** **Maximise Z = $\frac{x}{10} + \frac{3y}{20}$** **(Award full marks if the objective function is framed based on the total amount as Maximise Z = $\frac{11x}{10} + \frac{23y}{20}$.)**	**1**
	Writes the constraints of the given LPP as: $x + y \leq 20000$ $x \geq 5000$ $y \leq 8000$ $x, y \geq 0$	**1**

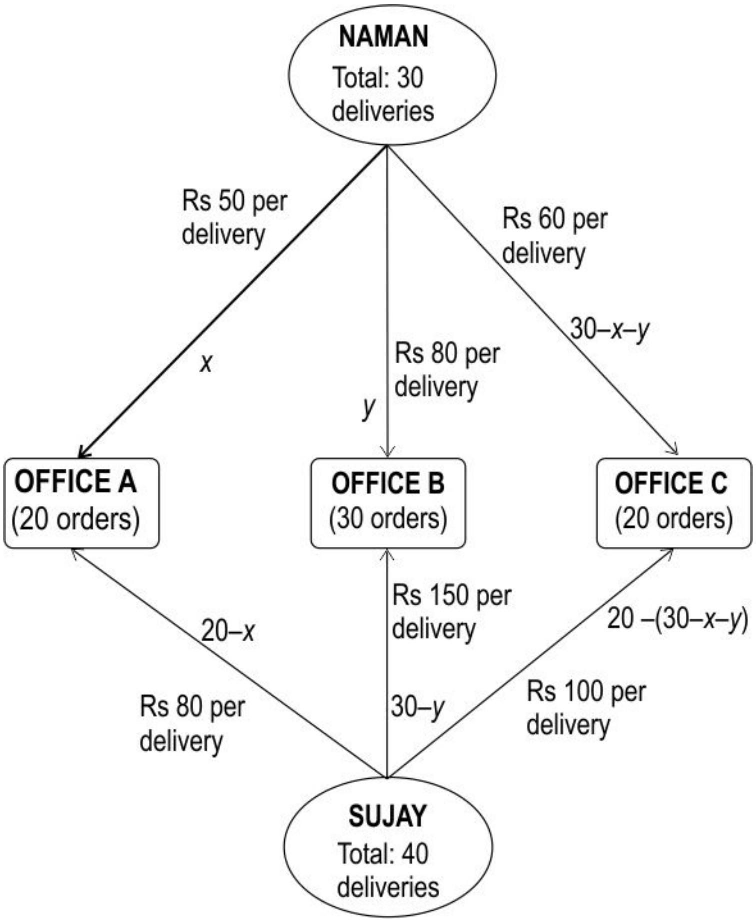
Q.No	What to look for	Marks
	Draws the graph of the system of inequalities and shades the feasible region. The graph may look as follows: 	**1.5**

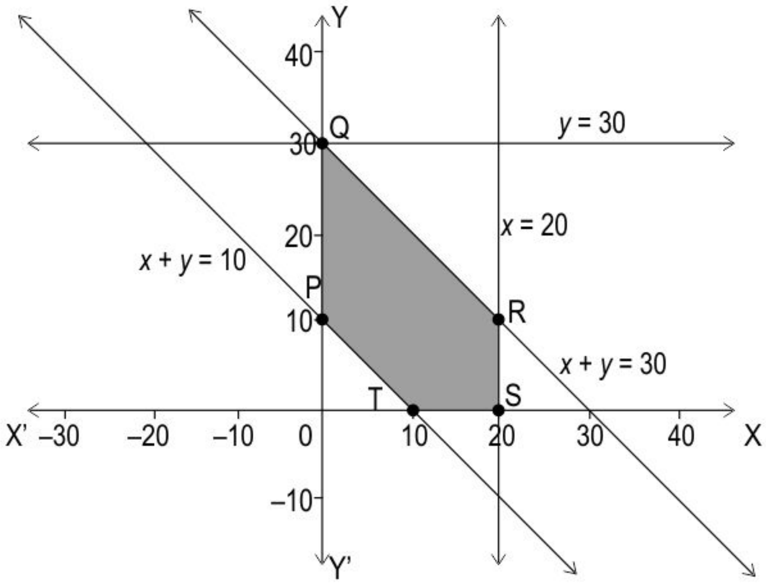
Q.No	What to look for	Marks
	Evaluates the objective function at the corner points to find the optimal solution as follows:	**1**

Corner points	**$Z = \frac{x}{10} + \frac{3y}{20}$**
A(5000, 0)	500
B(20000, 0)	2000
C(12000, 8000)	2400
D(5000, 8000)	1700

(Award full marks if correct Z values are obtained corresponding to the objective function $\frac{11x}{10} + \frac{23y}{20}$.)

	Writes that the maximum value of Z is obtained at the corner point C. Hence, concludes that (12000, 8000) is the optimal solution and Jayesh should invest Rs 12000 in bond X and Rs 8000 in bond Y.	**0.5**

Q.No	What to look for	Marks
10	**i) Considers *x* as the number of deliveries made by Naman to office A, and *y* as the number of deliveries made by Naman to office B.** **Forms the following inequalities:** $x \geq 0, y \geq 0$ $x + y \leq 30$ $x \leq 20$ $y \leq 30$ $x + y \geq 10$ **The completed tree diagram may look as follows:** 	1

Q.No	What to look for	Marks
	ii) Finds the delivery cost function, Z as 10 x - 30 y + 6900. **The working may look as follows:** **Delivery cost = 50 x + 80 y + 60(30 - x - y) + 80(20 - x) + 150(30 - y) + 100(x + y - 10)** **= 50 x + 80 y + 1800 - 60 x - 60 y + 1600 - 80 x + 4500 - 150 y - 1000 + 100 x + 100 y** **= 10 x - 30 y + 6900**	**1**
	iii) Plots lines x + y = 30, x = 20, y = 30 and x + y = 10 on a graph and finds the area enclosed by polygon PQRST as the feasible region. **The graph may look as follows:**  	**1**

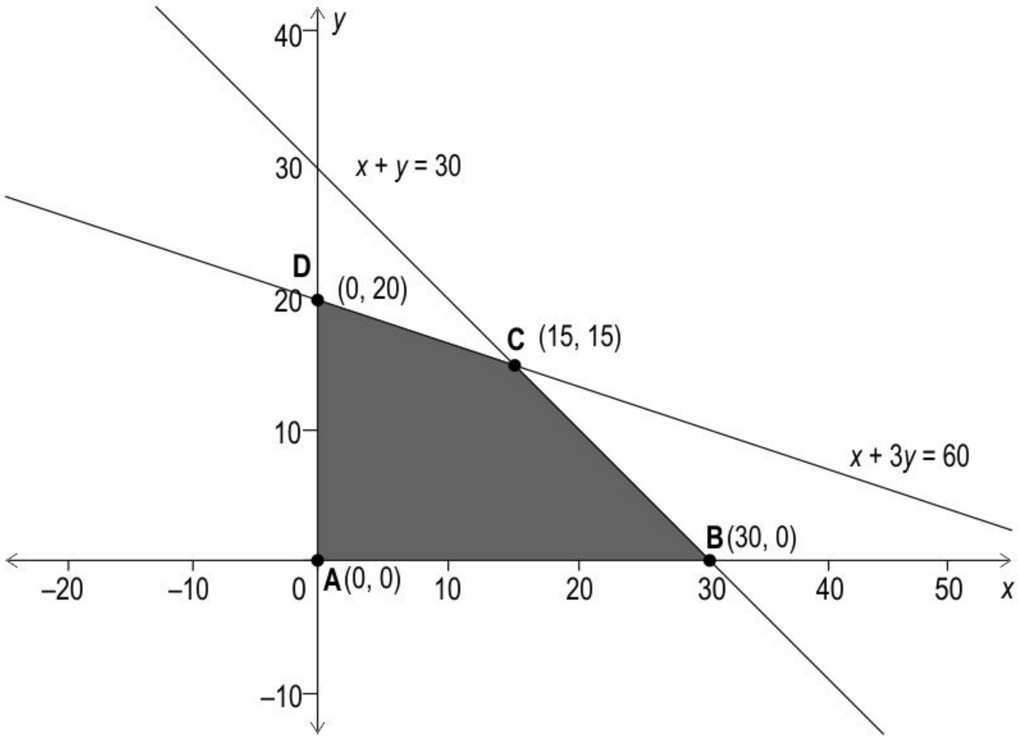
<table>
<tr><th>Q.No</th><th>What to look for</th><th>Marks</th></tr>
<tr><td></td><td>Finds the value of the cost function at each of the corner points as follows:

<table>
<tr><th>Point</th><th>x -coordinate</th><th>y -coordinate</th><th>Cost (Z)</th></tr>
<tr><td>P</td><td>0</td><td>10</td><td>Rs 6600</td></tr>
<tr><td>Q</td><td>0</td><td>30</td><td>Rs 6000</td></tr>
<tr><td>R</td><td>20</td><td>10</td><td>Rs 6800</td></tr>
<tr><td>S</td><td>20</td><td>0</td><td>Rs 7100</td></tr>
<tr><td>T</td><td>10</td><td>0</td><td>Rs 7000</td></tr>
</table></td><td>1</td></tr>
<tr><td></td><td>Finds that the cost is minimum when $x = 0$ and $y = 30$.

Writes that Naman should deliver 30 tiffins to office B, and Sujay should deliver 20 tiffins each to offices A and C.</td><td>1</td></tr>
<tr><td>11</td><td>Assumes the weight of premium nuts for each store as x kg and the weight of economy nuts for store A as y kg. And, writes the constraints of the given problem as:

$x + y \leq 30$
$x + 3y \leq 60$
$x \geq 0$
$y \geq 0$</td><td>1.5</td></tr>
<tr><td></td><td>Frames the objective function of the given problem as:

Maximize $Z = (80 \times 2x) + (50 \times 4y) = 160x + 200y$</td><td>0.5</td></tr>
</table>

Q.No	What to look for	Marks
	Draws the graph of the system of inequalities and shades the feasible region. The graph may look as follows: 	**1.5**
	Evaluates the objective function at the corner points to find the optimal solution as follows: Corner points: A(0, 0) → 0; B(30, 0) → 4800; C(15, 15) → 5400; D(0, 20) → 4000	**1**
	Writes that the maximum value of Z is obtained at the corner point C(15, 15) and concludes that the maximum profit of Rs 5400 is obtained when 15 kg of premium nuts are sold and 15 kg of economy nuts are sold.	**0.5**

Corner points	$Z = 160\,x + 200\,y$
A(0, 0)	0
B(30, 0)	4800
C(15, 15)	5400
D(0, 20)	4000